Control over Cavity Assisted Charging for Dicke Quantum Battery

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Abstract:
We study feed forward (open-loop) control approach for driving the cavity assisted charging process in the Dicke quantum battery, in which the coupling between the cavity and quantum two-level subsystem(s) plays a role of control parameter.The dynamics of the system is described with the Tavis - Cummings Hamiltonian. The analytical result is supported with the corresponding numerical simulations to demonstrate the high efficiency of the proposed control algorithm for charging Dicke quantum battery.

Keywords: Dicke quantum battery, Tavis - Cummings Hamiltonian, feedforward control.

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INTRODUCTION

Quantum Battery (QB), a quantum system based device, is capable to store and transfer the energy to other quantum devices, see the short review in [1]. Quantum batteries, as many other quantum systems, are expected to overcome their classical analogs in their efficiency due to the presence of quantum correlations[2,3]. Another quantum advantage is originated in the coherent cooperative interactions among the two-level quantum subsystems[4];it has been discovered and described in more details recently for Dicke QB [5]. From another hand, the QBs have an upper bound for the basic characteristics of the charging processes (the ergotropy, the charging power and others); the particular limit should be evaluated from the Fisher information and the energy variance of the battery [6].

In the present literature the variety of different physical realizations for quantum batteries has been described: Dicke QB, spin QB, harmonic oscillator QB [7,8]. Here we focus on the special type of Dicke quantum battery, where one cavity mode acting as charger is coupled to *N* qubits, which store the energy, see Figure 1. The two-level quantum system coupled to a cavity is widely represented in the literature[9]. This type of system can be extended for few cavities and few two-level sub-systems with different configuration of dynamical coupling [10]. The dissipative processes have drastic influence on the charging process[10,11].

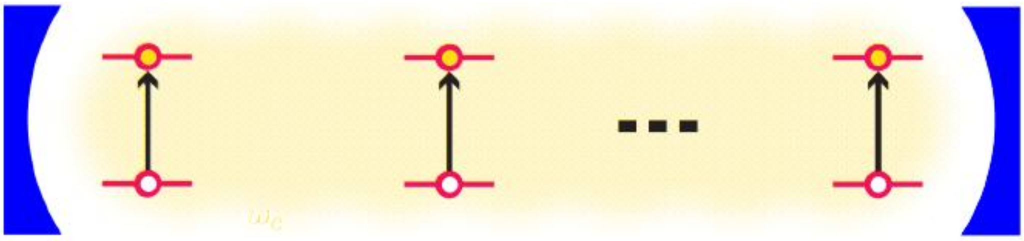

Figure 1.Schematic representation of a globally charging Dicke battery: many two-lewel quantum systems are coupled to the same cavity; Fig. based on [1].

Here we study different control approaches for driving Dicke QB during its charging process. The role of control parameter is played by the coupling between the cavity and quantum two-level subsystem(s). Although in the literature one can find studies on the control over cavity assisted charging, the types of the driving fields are limited usuallybyconstant control signals, rectangular pulses[12], monochromatic and bichromatic modulations[13]. Thus, in the terms of control theory we can find dominantly the feedforward (open-loop) algorithms with the very limited set of driving functions [14]. Our goal is to cover this lack and develop more variety of feedforward control fieldsto demonstrate their high efficiency for the cavity assisted charging in Dicke QB.

MODEL

We consider here the density of two-level quantum systems (batteries) not to be large: in this case we can apply the Dicke model Hamiltonian for our system[15].

The Tavis - Cummings model Hamiltonian

To model the control over the quantum battery coupled to a cavity, we use the non-resonant case of the Dicke model in the form of Tavis - Cummings Hamiltonian (the generalized Jaynes - Cummings case) [16]:

$$H = \omega_c \hat{a}^+ \hat{a} + \omega \cdot \hat{J}_z + \frac{1}{2}\omega_g u(t)\left[\hat{a}\hat{J}_+ + \hat{a}^+ \hat{J}_-\right]. \qquad (1)$$

Here the Plank constant $\hbar \equiv 1$; $\hat{a}^+, \hat{a}$ are the creation and annihilation operators for the single-mode cavity with the frequency ω_c; $\hat{J}_+, \hat{J}_-, \hat{J}_z$ are the components of the collective spin operators; ω is the energy level splitting of each two-level quantum system coupled to the cavity. The coupling parameter is time-depended and plays a role of control field in the model. We write in (1) a frequency dimension constant factor ω_g explicitly to get the control signal $u(t)$ as a function in the frequency-independent scale.

For the spin operators and the average $\langle \hat{a} \rangle = \alpha$ Eq.(1) allows the following semi-classical approximation[3,15]:

$$i\frac{dJ_-}{dt} = \omega \cdot J_- - 2\omega_g u\alpha \cdot J_z \ ;$$
$$i\frac{dJ_z}{dt} = \omega_g u \cdot \left(J_+\alpha - J_-\alpha^*\right); \qquad (2)$$
$$i\frac{d\alpha}{dt} = \omega_g u \cdot J_- + \omega_c \alpha \ .$$

The dynamical system (2) has an interesting property: it leads to a real solution to α[15]. Indeed, taking a new rotating basis $\alpha \to \alpha \exp\{-i\eta t\}$ and $J_- \to J_- \exp\{-i\eta t\}$ we get:

$$i\frac{dJ_-}{dt} = \tilde{\omega} \cdot J_- - 2\omega_g u\alpha \cdot J_z \ ;$$
$$i\frac{dJ_z}{dt} = \omega_g u\alpha \cdot \left(J_+ - J_-\right); \qquad (3)$$
$$i\frac{d\alpha}{dt} = \omega_g u \cdot J_- + \tilde{\omega}_c \alpha \ .$$

where $\tilde{\omega} = \omega - \eta$ and $\tilde{\omega}_c = \omega_c - \eta$.

Feedforward control for the creation – annihilation operators

First, let's apply the feedforward (open-loop) control procedure to the ctreation – annihilation operators' average: the third Eq. in the system (2). Using the ansatz based on the third equation from the system (3):

$$J_- = A(t)\alpha + iB(t)\frac{d\alpha}{dt} \ , \qquad (4)$$

with the real functions $A(t)$, $B(t)$, we obtain:

$$A(t) = -\frac{\tilde{\omega}_c}{\omega_g u(t)} \ ; \quad B(t) = \frac{1}{\omega_g u(t)} \ . \qquad (5)$$

Then we substitute (4)-(5) into the first equation of the system (3), and separate its real and imaginary part.For the imaginary part that implies:

$$\frac{dA}{dt}\cdot\alpha = (\tilde{\omega}B - A)\frac{d\alpha}{dt}\ , \qquad (6)$$

and by (5) finally:

$$\frac{1}{u}\frac{du}{dt} = \left(1+\frac{\tilde{\omega}}{\tilde{\omega}_c}\right)\frac{1}{\alpha}\frac{d\alpha}{dt}\ , \qquad (7)$$

with the solution:

$$\alpha(t) = \alpha(0) + [u(t) - u(0)]^{\tilde{\omega}_c/(\tilde{\omega}+\tilde{\omega}_c)}\ . \qquad (8)$$

Here we define the initial condition for the control signal:$u(0) = 0$.

CONTROL OVER THE STORED ENERGY

The energy W stored in Dicke QBs is associated with $\omega\cdot\hat{J}_z$ term of the Hamiltonian (1), rather than with whole internal Hamiltonian [1].

Feedforward control for thestored energy

The quasi-classical term $\omega\cdot J_z$ can be found from the real part of the first equation in (3) after the substitution of (4):

$$-B\cdot\frac{d^2\alpha}{dt^2} - \frac{dB}{dt}\cdot\frac{d\alpha}{dt} = \tilde{\omega}A\cdot\alpha - 2\omega_g u\cdot\alpha\cdot J_z\ . \qquad (9)$$

That implies for the stored energy:

$$W(t) = \omega\cdot J_z(t) = \frac{\omega}{2\omega_g u\cdot\alpha}\left(B\cdot\frac{d^2\alpha}{dt^2} + \frac{dB}{dt}\cdot\frac{d\alpha}{dt} + \tilde{\omega}A\cdot\alpha\right). \qquad (10)$$

By (5) and (7)we compute the second derivative for the operator average in the form:

$$\frac{1}{\alpha}\cdot\frac{d^2\alpha}{dt^2} = \frac{\tilde{\omega}_c}{\tilde{\omega}+\tilde{\omega}_c}\left(\frac{1}{u}\cdot\frac{d^2u}{dt^2} - \frac{\tilde{\omega}}{\tilde{\omega}+\tilde{\omega}_c}\frac{1}{u^2}\left(\frac{du}{dt}\right)^2\right). \qquad (11)$$

Expressing the stored energy W via the control signal $u(t)$, after simplification we get finally:

$$W(t) = \frac{\omega\tilde{\omega}_c}{2(\tilde{\omega}+\tilde{\omega}_c)\omega_g^2 u^4}\cdot\left[u\cdot\frac{d^2u}{dt^2} - \frac{2\tilde{\omega}+\tilde{\omega}_c}{\tilde{\omega}+\tilde{\omega}_c}\cdot\left(\frac{du}{dt}\right)^2 - \tilde{\omega}(\tilde{\omega}+\tilde{\omega}_c)\cdot u^2\right]. \qquad (12)$$

Eq.(12) solves the general problem of open-loop control: it represent the stored energy in the Dicke battery via the arbitrary feedforward control function $u(t)$explicitly inthe form of nonlinear differential equation of the second order. We remind here that this function reflects the controlled interaction between the cavity and the storing two-level quantum systems.

Stabilization of the stored energy

First of all, we need to mention that for some certain cases of $u(t)$ Eq.(12) is divergent. It means that the ansatz model (4) is not valid for such shapes of control.

To stabilize the energy W stored in the Dicke QB at the certain target level W_*, we demand the function in RHS(12) to come to some saturation level. This is a typical mathematical problem for the open-loop control: defining the solution of the equation (12), we need to search for an appropriate control signal $u(t)$.

Differential Eq.(12) is nonlinear, and we cannot find its general solution for $u(t)$. In this case we could guess a reasonable shape of the control signal with a free parameter, and we need to find the value of this parameter from the numerical analysis of our dynamical system.

Numerical simulations

For real solid state devices the typical frequencies ω and ω_c are ranged from GHz up to THz,while for the coupling constant $\omega_g$we can accept the interval 10 – 100 MHz [12]. The phase shift η in (3) can be taken as 0.For our numerical simulations we guess: $\omega = \tilde{\omega} = \tilde{\omega}_c = 10^9$; $\omega_g = 10^7$.

For the control signal we chose the saturation function:

$$u(t) = u(0)e^{-t/T} + u_*(1 - e^{-t/T}) \,, \qquad (13)$$

with $T = 10$, starting from the large pumping of the energy to the system $u(0)=50$. The result of numerical simulations for the stored energy and the control field are represented on Figure 2.

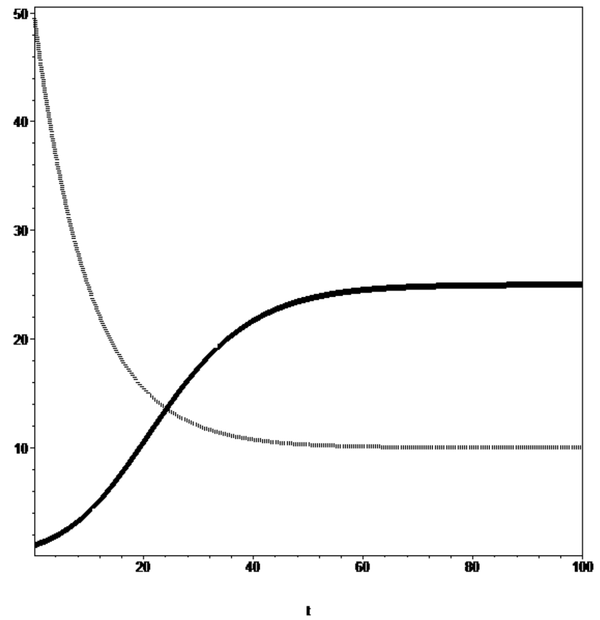

Figure 2. Stabilization of the stored energy $W(t)$ (solid line) under the control field $u(t)$ (thin dot line) at the level W_*= 25.

We seefrom the plot on Figure 2 that for the stabilization of the stored energy at the level 25 we need to choose $u_* = 10$. In general, we can find the corresponding u_* for each level of the stored energy stabilization. The control is decreasing in time, i.e., the rate of the energy pumped into the system to be stored is transferred mostly at the early stages of control.

CONCLUSIONS

The feedforward (open-loop) algorithm proposed in the paper demonstrates the achievability of the stabilization for the goal energy value $W(t)$ in the charging process of Dicke battery. Thus, the standard set of open-loop algorithms beyond the simplest rectangular/harmonic shape of the control could be successfully applied for the cavity assisted charging of QB.The numerical simulations for our model present the results in a good accordance with our analytical analysis. Nevertheless, the open-loop approach application is very limited for the general modeling of all stages of QB

working: charging, effective storage of the energy, transfer the stored energy to other quantum devices.
For that reason at the further steps we plan to study different alternative feedback algorithms, in Pontryagin's optimal [17] and Fradkov's[14]and Kolesnikov's[18,19]sub-optimal forms. The priority of last two algorithms is originated intheir high robustness, their non-sensitivity towards the relatively small perturbations of a quantum dynamical system and in the possibility of multi-goal reformulation for many quantum systems [20,21], including QBs [22]. The same approach, we strongly believe, will work for the controlled quantum batteries as well.

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